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INVERSE DOMINATION IN SOME OPERATIONS ON INTERVAL-VALUED FUZZY GRAPHS

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Abstract:

The inverse dominance number □□(□) was invented and examined in this paper for several operations on interval-valued fuzzy graphs, such as union, join, and composition.

. **Keywords:** Interval-valued fuzzy graph, Inverse domination number, Operation on fuzzy graph, Inverse domination set.

1. Introduction

In the discipline of mathematics, a fuzzy graph is an application tool that allows users to concisely define the relationship between any two concepts. As an extension, Zadeh[16] developed the notion of interval-valued fuzzy sets. Of fuzzy sets [17], where the membership degrees' values are intervals of instead of the numbers, we'll use the numbers. Traditional fuzzy sets do not adequately describe uncertainty, however interval-valued fuzzy sets do. The hazy relationships Rosenfeld [11] also explored the relationship between fuzzy sets, and he derived the fuzzy graphs have a specific structure. Recently. Several characteristics and operations on interval-valued fuzzy graphs have been examined by Akram and Dudek [3]. Several significant works [4, 5, 6, 7, 8, 9, 10, 15] can be found in fuzzy graph theory.

2. Preliminaries

We review in this section, some basic definitions related to interval-valued fuzzy graphs and domination in fuzzy graph.

An interval-valued fuzzy graph of a graph $G^* = (V,E)$ is a pair $G = (A,B)$, where $A = [\mu^-_A,\mu^+_A]$ is an interval-valued fuzzy set on V, $B = [\rho^-_B,\rho^+_B]$ is an interval-valued fuzzy relation on V, such that $\mu^-_A(x) \leq \mu^+_A(x);\forall x \in V$ and $\rho^-_B(x,y) \leq min\{\mu^-_A(x),\mu^-_A(y)\}$ and $\rho^+_B(x,y) \leq min\{\mu^+_A(x),\mu^+_A(y)\};\forall(x,y) \in E$. In an interval-valued fuzzy graph G, when $\rho^-(u,v) = \rho^+(u,v) = 0$ for some u and v, then there is no edge between u and v. Otherwise there exists an edge between u and v. Let $G = (A,B)$ be an interval-valued fuzzy graph. Then the cardinality of interval-valued fuzzy graph G is defined as

$$|G| = \sum_{u\in V} \frac{1+\mu^+(u)-\mu^-(v)}{2} + \sum_{(u,v)\in E} \frac{1+\rho^+(u,v)-\rho^-(u,v)}{2}$$

The vertex cardinality of an interval-valued fuzzy graph G is defined by

$$|V| = \sum_{u\in V} \frac{1+\mu^+(u)-\mu^-(u)}{2}$$

.

For all $u \in V$ is called the order of an interval-valued fuzzy graph and is denoted by $p(G)$. The edge cardinality of an interval-valued fuzzy graph G is defined by

$$|E| = \sum_{(u,v)\in E} \frac{1+\rho^+(u,v)-\rho^-(u,v)}{2}$$

.

For all $(u,v) \in E$ is called the size of an interval-valued fuzzy graph and is denoted by $q(G)$. An edge $e = (x,y)$ of an interval-valued fuzzy graph is called effective edge if $\rho^+(x,y) = min\{\mu^+(x),\mu^+(y)\}$ and $\rho^-(x,y) = min\{\mu^-(x),\mu^-(y)\}$. The degree of a vertex can be generalized in different ways for an interval-valued fuzzy graph $G = (A,B)$. The effective degree of a vertex v in an interval-valued fuzzy graph, $G = (A,B)$ is defined to be summation of the weights of the effective edges incident at v and it is denoted by $d_E(v)$. The minimum effective edges degree of G is $\delta_E(G) = min\{d_E(v)|v \in V\}$. The maximum effective degree of G is $\Delta_E(G) = max\{d_E(v)|v \in V\}$. A vertex u of an interval-valued fuzzy graph G is said to be an isolated vertex if $\rho^-(uv) < min\{\mu^-(u),\mu^-(v)\}$ and $\rho^+(uv) < min\{\mu^+(u),\mu^+(v)\}$ for all $v \in V - \{u\}$ such that there is an edge between u and v, i.e., $N(u) = \varphi$. A set S of vertices of an interval-valued fuzzy graph G is said to be independent if $\rho^-(uv) < min\{\mu^-(u),\mu^-(v)\}$ and $\rho^+(uv) < min\{\mu^+(u),\mu^+(v)\}$ for all $u,v \in S$. An interval-valued fuzzy graph, $G = (A,B)$ is said to be Complete interval-valued fuzzy graph if $\rho^-(v_i,v_j) = min\{\mu^-(v_i),\mu^-(v_j)\}$, $\rho^+(u,v) = min\{\mu^+(u),\mu^+(v)\}$, for all $u,v \in V$ and denoted by K_p. The complement of an interval-valued fuzzy graph, $G = (A,B)$ is an interval-valued fuzzy graph, $G = (A,B)$ where

$\overline{\rho^-}(u,v) = min\{\mu^-(u),\mu^-(v)\} - \rho^-(u,v)$ and $\overline{\rho^+}(u,v) = min\{\mu^+(u),\mu^+(v)\} - \rho^+(u,v)$ for all $u,v \in G$. An interval-valued fuzzy graph $G = (A,B)$ of a graph $G^* = (V,E)$ is said to be bipartite if the vertex set V can be partitioned into two nonempty sets V_1 and V_2 such that $\rho^-(xy) = 0$ and $\rho^+(xy) = 0$ if $x,y \in V_1$ or $x,y \in V_2$. Further if $\rho^-(xy) = min\{\mu^-_A(x),\mu^-_A(y)\}$ and $\rho^+(xy) = min\{\mu^+_A(x),\mu^+_A(y)\}$ for all $x \in V_1$ and $y \in V_2$. Then G is called a complete bipartite fuzzy graph is denoted by $K_{\mu}-A,\mu+A$, where μ^-_A and μ^+_A are restrictions of μ^-_A and μ^+_A on V_1 and V_2 respectively. An edge $e = xy$ of an interval-valued fuzzy graph G is called an effective edge if $\rho^-(xy) = min\{\mu^-_A(x),\mu^-_A(y)\}$ and

$$\rho^+(xy) = min\{\mu^+_A(x),\mu^+_A(y)\}.$$

In this case, the vertex x is called a neighbor of y and conversely. $N(x) = \{y \in V : y$ is a nieghbour of $x\}$ is called the neighborhood of x. A subset $(S \subseteq V)$ of $V(G)$ is called dominating set in interval-valued fuzzy graph G, if for every $v \in V - S$ there exists $u \in S$ Such that (u,v) is strong edge. A dominating set S of G is called minimal dominating set of G if $S - \{u\}$ is not dominating set for every $u \in S$. A minimal dominating set S, with $|S| = \gamma(G)$ is denoted by $\gamma-set$. Let $G = (A,B)$ be an interval-valued fuzzy graph and let S be $\gamma - set$ of G, if $V - S$ has a dominating set S^8. Then S^8 is called an inverse dominating set of G with respect to S. An inverse dominating set S^8 of interval valued fuzzy graph G, is called minimal inverse dominating set, if $S^8 - \{u\}$ is not inverse dominating set, for every $u \in S^8$. The minimum fuzzy cardinality taken over all minimal inverse dominating sets of an interval-valued fuzzy graph G, is called The inverse domination number of an interval-valued fuzzy graph G and denoted by $\gamma^8(G)$ or simply γ^8. The maximum fuzzy cardinality among all minimal inverse dominating set of an interval-valued fuzzy graph G is called the upper inverse domination number of G and is denoted by $\Gamma^8(G)$.

3. Results & discussion

Theorem 3.1. Let G_1 and G_2 be two vertices disjoint interval-valued fuzzy graphs, then $\gamma^8(G_1 \cup G_2) = \gamma^8(G_1) + \gamma^8(G_2)$.

Proof. Let S_1 and S_2 be tow γ_1−set and γ_2 of G_1 and G_2 respectively. Since

G_1 and G_2 are vertices disjoint. By Theorem (1) [2] and Theorem(2) [1], we get

$$\gamma^8(G_1 \cup G_2) \leq |V_1 + V_2 - S_1 \cup S_2)| = p_1 - \gamma(G_1) + p_2 - \gamma(G_2) = \gamma^8(G_1) + \gamma^8(G_2). \square$$

In the flowing Example, we discuss in detail the result in the above Theorem.

Example 3.2. Consider the interval-valued fuzzy graphs G_2 and G_1 given in the Figures 5.1*a* and

5.1*b*, respectively, such that all edges of G_1 and G_2 are strong edges.

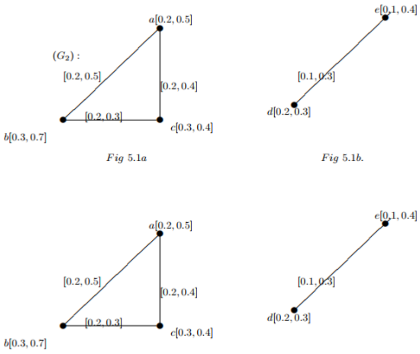

Fig 5.1a *Fig 5.1b.* *Fig 5.1c* : $(G_1 \cup G_2)$

.

The union of G_1 and G_2 is shown in Figure (5.1*c*,) we see that $S_1^8 = \{b,c\}$ is a $\gamma^8 -$ *set* of G_1, $S_2^8 = \{e\}$ is a $\gamma^8 -$ *set* of G_2 and $\gamma_1^8 = 1.25, \gamma_2 = 0.65$. Now, $S^8 = S_1^8 \cup S_2^8$ is a γ^8–*set* of $G_1 \cup G_2$. Hence $\gamma^8(G_1 \cup G_2) = |S_1 \cup S_2| = |S_1 + S_2| = 1.25 + 0.65 = 1.9 = \gamma_1 + \gamma_2$.

The flowing Theorem given γ^8 of $G_1 + G_2$.

Theorem 3.3. Let G_1 and G_2 be any two interval-valued fuzzy graphs, then

$$\gamma^8(G_1+G_2) = min\{\gamma^8(G_1), \gamma^8(G_2)\}.$$

Proof. Let G_1 and G_2 be two interval-valued fuzzy graphs with γ_1^8 and γ_2^8 respectively.Then by Remark(3.1),[1]

$$\gamma^8(G_1 + G_2) \le p - \gamma(G_1 + G_2)$$

Since the inverse dominating set is dominating set. Then be Theorem (4) [2] and Definitions (3.1),(3.2), (3.3) and Definition of Join, [1] we get

$$\gamma^8(G_1 + G_2) = min\{\gamma^8(G_1, \gamma^8 G_2)\}.$$

The flowing example discuss in detail the result in the above Theorem.

Example 3.4. Consider the interval-valued fuzzy graphs G_1 and G_2 given in the Figures 5.2*a* and 5.2*b* respectively, such that all edges in G_1 and G_2 are effective

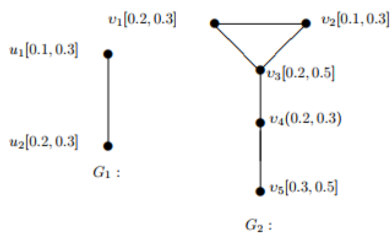

Fig 5.2a

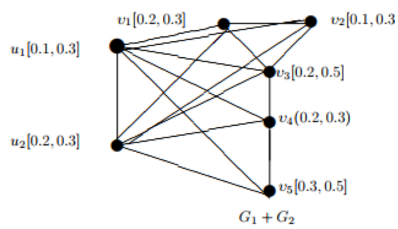

Fig 5.2b

The join of G_1 and G_2 given in Figure (5.2*b*) we can verify that $|S_1^8| = |\{u_1\}| = 0.60$ is a γ^8–*set* of G_1, $|S_2^8| = |\{v_1, v_5\}| = 1.2$ is a γ^8–*set* of G_2. Hence

$$\gamma^8(G_1 + G_2) = min\{S_1^8, S_2^8\} = \{u_1\} = 0.60$$

The flowing Theorem given γ for the composition of any tow interval-valued fuzzy graphs G_1 and G_2.

Theorem 3.5. Let S_1^8 and S_2^8 be $\gamma^8 -$ *sets* respect to

□

S_1 and S_2 of intervalvalued fuzzy graphs G_1 and G_2, respectively. Then

$$\gamma(G_1 \circ G_2) = min\{|S_1^8 \times S_2|, |S_2^8 \times S_1|\}.$$

Proof. Let $(u,v) \in S_1^8 \times S_2$.: Since S_1^8 is γ^8 –*set* respect to S_1 of G_1, Therefore,

S_1^8 is $\gamma -$ *set* of G_1

Similarly, $(u,v) \in S_2^8 \times S_1$. Now: By Theorem (8),[2] we get: $|S_1^8 \times S_2|$ and $|S_2^8 \times S_1|$ are $\gamma - sets$ of $(G_1 \circ G_2)$

Since $\gamma(G_1 \circ G_2) = |S_1 \times S_2$ Theorem (8) [2]. Then $|S_1^8 \times S_2|$ and $|S_2^8 \times S_1|$ are $\gamma^8 - sets$ of $(G_1 \circ G_2)$

Hence, by Definitions (3.1), (3.5),[1] and Definition of composition Hence,

$\gamma(G_1 \circ G_2) = min\{|S_1^8 \times S_2|,|S_2^8 \times S_1|\}$.

The flowing example explain the results in above Theorem.

Example 3.6. For the interval-valued fuzzy graphs G_2 and G_1 given in the Figures 5.4*a* and 5.4*b*, respectively.

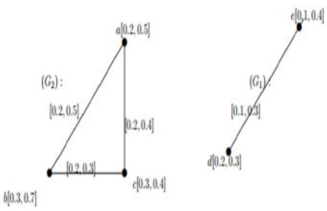

Fig 5.4a Fig 5.4b

We can verify that $S_1^8 = \{e\}$ is γ^8–set respect to $S_1 = \{d\}$ of G_1 and $S_2^8 = \{b,d\}$ is γ^8–set respect to $S_2 = \{a\}$ of G_2

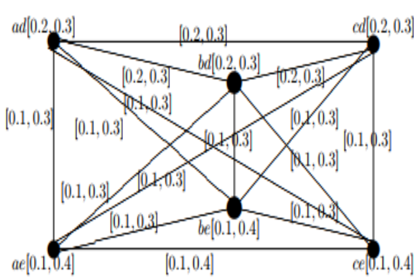

Fig 5.4c : $G_1 \circ G_2$

We can verify that $S_1^8 \times S_2 = \{ae\}$ and $S_2^8 \times S_1 = \{db,dc\}$ are a γ^8–sets of $G_1 \circ G_2$. Hence $\gamma G_1 \circ G_2 = min\{|S_1^8 \times S_2|,|S_2^8 \times S_1|\} = min\{|ae|,|bd,dc|\} = min\{0.65,1.1\} = 0.65$

4. Conclusion

In this paper, the concepts of inverse domination number were found of some operations on interval-valued fuzzy graphs and were discussed with the suitable examples.

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